

Speed Math Solutions

CHILES MINI MU

2022-2023

1. Find the remainder when 123456789 is divided by 99.

Solution. Let $n = 123456789$ for brevity. Then

$$n \equiv 1 + 2 + \cdots + 9 \equiv 0 \pmod{9}$$

and

$$n \equiv 1 - 2 + 3 - \cdots + 9 = 5 \pmod{11}.$$

Combining the two relations gives $n \equiv \boxed{27} \pmod{99}$. \square

2. Shaoyang can buy dog treats in groups of 20 treats, 23 treats, or 2023 treats. What is the largest amount of dog treats that he cannot buy?

Solution. The 2023 is too large to be relevant; then from Chicken McNugget Theorem the answer is just $20 \cdot 23 - 20 - 23 = \boxed{417}$. \square

3. Find the sum of all prime numbers less than 23.

Solution. The sum of all prime numbers up to 23 is 100, so the answer is just $100 - 23 = \boxed{77}$. \square

4. Nelson is taking turns shooting free throws with LeBron James, star of the popular movie Space Jam: A New Legacy. Nelson has a $\frac{1}{10}$ chance of making a free throw, and LeBron has a $\frac{3}{4}$ chance of making a free throw. Given that Nelson goes first, what is the probability that he makes a free throw before LeBron does?

Solution. This probability is given by the geometric series with first term $\frac{1}{10}$ and common ratio $(1 - \frac{1}{10})(1 - \frac{3}{4}) = \frac{9}{40}$, so the answer is $\frac{1}{10} \div (1 - \frac{9}{40}) = \boxed{\frac{4}{31}}$. \square

5. What is the smallest possible value of $x^2 + 6x + 24$ for real x ?

Solution. Note that $x^2 + 6x + 24 = (x + 3)^2 + 15$, so the answer is $\boxed{15}$. \square

6. Compute $12 \times 13 - 20 + 3(5 + 36 \div 4)$.

Solution. This equals

$$156 - 20 + 3(5 + 9) = 136 + 3 \cdot 14 = 136 + 42 = \boxed{178},$$

as desired. \square

7. The four digit number $\overline{202A}$ is divisible by 23. What is the remainder when this number is divided by 22?

Solution. From long division, we have $2020 = 87 \cdot 23 + 19$, so the number must be $88 \cdot 23$, which is actually divisible by 22, so the answer is $\boxed{0}$. \square

8. Compute $214 \cdot 226$.

Solution. We have

$$214 \cdot 226 = (220 - 6)(220 + 6) = 220^2 - 6^2 = 48400 - 36 = \boxed{48364},$$

as desired. \square

9. Given that a cylinder with radius 4 has equal volume and surface area, what is its height?

Solution. If we let h denote the height of the cylinder, we have

$$32\pi + 8h\pi = 16h\pi \implies 8h\pi = 32\pi \implies h = \boxed{4},$$

as desired. \square

10. Let $f(x) = x^2 - 20x + 23$. Compute $f(19) + f(50)$.

Solution. Note that $f(x) = (x - 10)^2 - 77$, so

$$f(19) + f(50) = 9^2 + 40^2 - 2 \cdot 77 = 41^2 - 154 = 1681 - 154,$$

so the answer is $\boxed{1527}$. \square

11. Khawla and Linda are playing a game. Khawla starts with a positive integer, doubles it, and then passes it to Linda. Linda adds 3 to the number and passes it back to Khawla. Khawla doubles it again, passes it to Linda, who again adds 3 to it and passes it back, and so on. Find the sum of all possible starting numbers given that the number equals 69 at some point.

Solution. Note that if 69 were in the sequence of numbers, it must have been the result of adding 3 to a number and not being doubled (since it's odd). Then working backwards, we can construct the sequence 69, 66, 33, 30, 15, 12, 6, 3, at which point we stop since 3 is odd. Since Khawla starts, the answer is just $33 + 15 + 6 = \boxed{54}$. \square

12. Find the number of distinguishable permutations of *ROBSNOW*.

Solution. This is just $7!/2! = 5040/2 = \boxed{2520}$. \square

13. Given that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots = \frac{\pi^2}{6}$, compute $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots$, where the terms in the sum are the reciprocals of the odd perfect squares.

Solution. Note that

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \sum_{n=1}^{\infty} \frac{1}{n^2} - \sum_{n=1}^{\infty} \frac{1}{(2n)^2} = \frac{3}{4} \sum_{n=1}^{\infty} \frac{1}{n^2} = \boxed{\frac{\pi^2}{8}},$$

where the last equality follows from the given. \square

14. Find the side length of the equilateral triangle that has equal area and perimeter.

Solution. Let s denote the side length of the triangle. Then $3s = s^2\sqrt{3}/4$, and solving gives $s = \boxed{4\sqrt{3}}$. \square

15. Right before Linsey goes to sleep, she looks at a clock which reads 2 : 49. She wakes up in the middle of the night and looks at the clock again, and surprisingly, the product of the digits shown on the clock are the same as they were right before she went to sleep. What is the least amount of sleep she could have possibly gotten (in minutes)?

Solution. Note that the initial product of digits is $2 \cdot 4 \cdot 9 = 72$. If she woke up before 3 : 00, then one of the digits would be 5, which does not divide 72, so she must have waken up after 3 : 00. From here, we clearly have that the earliest possible times she could have waken up are 3 : 38 and 3 : 46, the first of which yields an answer of $\boxed{49}$ minutes. \square

16. Find the units digit of $20^{23} + 23^{20} + 2023^{2023}$.

Solution. This is just $0 + 1 + 7 = \boxed{8}$. \square

17. Wesley flips 6 fair coins and then rolls a fair die. What is the probability that the number he rolls on the die equals the number of coins which turn up heads?

Solution. Regardless of the results of the coin flips, the die has a $\frac{1}{6}$ chance of returning the correct number, so the answer is just $\boxed{\frac{1}{6}}$. \square

18. Find the number of ordered pairs (a, b) of positive integers satisfying $a^2 + b^2 = 699$.

Solution. Note that $699 \equiv 3 \pmod{4}$, and perfect squares are either 0 or 1 modulo 4, so the answer is just $\boxed{0}$. \square

19. Cyrus, Yimo, and Jiayi are swimming around a circular pool. It takes them 289 seconds, 170 seconds, and 119 seconds, respectively, to travel one time around the pool. Given that they start at the same point and travel in the same direction, what is the least amount of time (in seconds) until all three of them are at the same point again?

Solution. The answer is just the least common multiple of $289 = 17^2$, $170 = 2 \cdot 5 \cdot 17$, and $119 = 7 \cdot 17$, which is $2 \cdot 5 \cdot 7 \cdot 17^2 = \boxed{20230}$. \square

20. Evaluate $20(2 + 3(2 + 0(2 + 3) + 20 + 23))$.

Solution. Working from the inside out, this is equal to

$$20(2 + 3(2 + 20 + 23)) = 20(2 + 3 \cdot 45) = 20 \cdot 137 = \boxed{2740},$$

as desired. \square

21. Find the area of the triangle with vertices at $(2, 2)$, $(2, 3)$, and $(20, 23)$.

Solution. The distance from $(20, 23)$ to the base formed by the other two points is $20 - 2 = 18$, and this base has length $3 - 2 = 1$, so the area is just $\frac{1}{2} \cdot 18 \cdot 1 = \boxed{9}$. \square

22. James falsely believes that adding 8 to any odd composite number will yield a prime number. Find the smallest possible odd composite number which is a counterexample to James's belief.

Solution. The first several odd composite numbers are 9, 15, 21, 25, 27, 33. We see that the first pair which differs by 8 is 25, 33, so the answer is $\boxed{25}$. \square

23. Find the smallest positive integer n such that the sum of the angles in an n -gon is greater than 2023° .

Solution. We have

$$180(n - 2) > 2023 \implies n - 2 \geq 12 \implies n \geq 14,$$

so the answer is $\boxed{14}$. □

24. Ryan gets 7 nickels for every math problem he solves correctly. If Ryan gets \$1337 from only these nickels, how many math problems did he solve correctly?

Solution. This is just $(1337 \cdot 100)/(7 \cdot 5) = 191 \cdot 20 = \boxed{3820}$. □

25. Simplify $\sqrt{1!} + \sqrt{2!} + \sqrt{3!} + \sqrt{4!} + \sqrt{5!}$.

Solution. This equals $1 + \sqrt{2} + \sqrt{6} + 2\sqrt{6} + 2\sqrt{30} = \boxed{1 + \sqrt{2} + 3\sqrt{6} + 2\sqrt{30}}$. □